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SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered) 20. ABSTRACT (Continued) second plate is backed by water and air are computed using both methods. The lumped-parameter model of the interior fluid is found to degrade the detail of the responses of the plates compared to the continuous-fluid solution, but the general features of the response (timing and magnitude of the peak velocities, for example) are not greatly affected even for relatively crude representations of the fluid, of two to five fluid elements. The calculations are carried out to better understand the interaction of a shockwave from an underwater explosion and the sail appendage of a submarine.

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THE RESPONSE OF TWO FLUID-COUPLED PLATES TO AN INCIDENT PRESSURE PULSE

INTRODUCTION

Investigations of the interaction of plates and pressure waves have been used to predict the effects of underwater explosions on a given structure [1-3]. These predictions aided in designing structures capable of resisting underwater explosions. The studies relied on flat-plate theory and involved a single air or water-backed plate. The fluid involved was an acoustic fluid (irrotational, inviscid, with speed of sound independent of pressure). The present investigation differs only from those above in that two flat plates coupled by an acoustic fluid are considered. The case where the second plate is water-backed models the sail appendage of a submarine. The case where the second plate is air-backed is also computed.

Another class of investigations consists of rigorous treatments of the interaction of a shock wave with an elastic cylindrical or spherical shell, or concentric shells which are fluid coupled [4-9]. These studies incorporate radiation, scattering, diffraction effects and elastic deformation of the shell. The cases considered in this report are not of the type above. The plates are infinite, free, perfectly rigid and are excited by a plane wave pressure pulse of infinite extent. The only motion is the translation of the plate in the direction normal to the plate. An exposition of flat-plate theory can be found in Cole or Keil [10] [11].

FLAT-PLATE CALCULATION WITH CONTINUOUS FLUID

As a first case consider two infinite free plates with mass per unit area $\rm m_1$ and $\rm m_2$ and water in the three regions as depicted in Figure 1. The plates are separated by a distance A. The pressures shown represent the incident, transmitted, and reflected wave system. The equations of motion are from flatplate theory:

$$m\ddot{x}_1 = P_1 + P_2 - P_3 - P_4, \qquad x_1(0) = \dot{x}_1(0) = 0$$
 (1)

$$m_{\tilde{\chi}_{2}}^{*} = P_{5} + P_{6} - P_{7}, \qquad X_{2}(0) = \tilde{\chi}_{2}(0) = 0$$
 (2)

where the pressures are

$$P_1 = P_0 e^{-st}$$
 (3)

$$P_2 = P_1 - oc\dot{x}_1(t)$$
 $P_3 = oc\dot{x}_1(t) + P_4(t)$
 $P_4 = P_6(t - A/c)$

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$$P_5 = P_3(t - A/c)$$

$$P_6 = P_5 - oc \dot{x}_2(t)$$

$$P_7 = pc\dot{x}_2(t)$$

and where α is the density of seawater, about 1026 kg/m³, c is the speed of sound in seawater about 1528m/s₁, and s is the decay constant for the pressure pulse, $1000 \, s^{-1}$.

The equations with $m_1=m_2=m$ may be written for the first three time intervals, 0 to A/c, A/c to 2A/c, and 2A/c to 3A/c as:

$$\ddot{X}_1 + \frac{2\rho c}{m}\dot{X}_1 = \frac{2P_0}{m} e^{-st}$$
 (5)

$$\ddot{x}_2 = 0$$

A/c to 2A/c

$$\ddot{\chi}_1 + \frac{2\rho c}{m}\dot{\chi}_1 = \frac{2P_0}{m} e^{-st} \tag{6}$$

$$\ddot{x}_{2} + \frac{2\rho c}{m}\dot{x}_{2} = \frac{2\rho c}{m}\dot{x}_{1}(t - A/c)$$
 (7)

2A/c to 3A/c

$$\ddot{x}_1 + \frac{2\rho c}{m}\dot{x}_1 = \frac{2Po}{m}e^{-st} + \frac{2\rho c}{m}\dot{x}_2(t-A/c) - \frac{2\rho c}{m}\dot{x}_1(t-2A/c)$$
 (8)

$$\ddot{x}_2 + \frac{2\rho c}{m}\dot{x}_2 = \frac{2\rho c}{m}\dot{x}_1(t-A/c)$$
 (9)

The solutions for the velocities may be obtained by standard methods for solving differential equations. They are for the three time periods:

0 to A/C

$$\dot{X}_1 = \frac{2P_0}{m(s-r)} [e^{-rt} - e^{-st}], r = 2\rho c/m$$
 (10)

$$\dot{x}_2 = 0 \tag{11}$$

A/C to 2A/C

$$\dot{X}_1 = \frac{2P_0}{m(s-r)} [e^{-rt} - e^{st}]$$
 (12)

$$x_{2} = \frac{2rP_{0}}{m(s-r)}t^{2}e^{-rt^{2}} - \frac{2rP_{0}}{m(s-r)^{2}}[e^{-rt^{2}}-e^{-st^{2}}], t^{2} = t - A/c$$
 (13)

2A/c to 3A/c

$$\dot{x}_1 = \frac{r^2 P_0}{m (s-r)} t^2 e^{-rt} - \frac{2r^2 P_0}{m (s-r)^2} t^2 e^{-rt} + \frac{2r^2 P_0}{m (s-r)^3} [e^{-rt} - e^{-st}]$$

$$-\frac{2rP_0}{m(s-r)}$$
 t'e^{-rt'} + $\frac{2rP_0}{m(s-r)}$ 2 [e^{-rt'} - e^{-st'}]

$$+\frac{2P_0}{m(s-r)}[e^{-rt}-e^{-st}], t'=t-2A/C$$
 (14)

$$\dot{x}_2 = \frac{2rP_0}{m(s-r)} t'e^{-rt'} - \frac{2rP_0}{(s-r)^2} [e^{-rt'} - e^{-st'}], t' = t - A/C$$
 (15)

Consider the situation where the plates are of different masses and the second plate is air-backed. The equations of motion change slightly in three time periods considered, since p = 0 and m \neq m $_2$. The solutions to these equations are similar to the water-backed case.

Figure 2a shows numerical results for the first case where the second plate is water-backed. The mass of both plates is 124 kg/m² and the pressure pulse has an initial pressure of $P_{o} = \rho cu$ with u_{o} the particle velocity equal to lm/s. The time A/c is about 2.1 ms. At t = 0 the first plate begins accelerating, reaches a peak velocity, and then decelerates in a typical exponential fashion. At t = A/c the transmitted wave reaches the second plate and accelerates the second plate to very nearly the velocity of the first plate. At t = 2A/c, the wave reflected from plate 2 reaches plate 1 and imparts to it a small negative velocity. Figure 2b shows the time history of the pressures on the front and back of plate 1. The pressure on the front of plate I leads in time the pressure on the back of plate 1 but when the reflected wave returns to plate 1 the pressure on the back leads the pressure on the front. The pressure on the front of the second plate leads in time the pressure on the back as can be seen in Figure 2c. At both plate 1 and 2, the pressures on the front and back cross at the time when the peak velocities occur.

Figure 3a shows the velocities of the plates for the case where the second plate is air-backed, the other parameters being the same as the previous case. The first plate reaches about the same velocity as in the first case. The second plate being air-

backed reaches a velocity almost double that of the water-backed plate. The reflected wave imparts a small negative velocity to plate 1 at t = 2A/c, however, unlike the water=backed case, plate 1 experiences a second large forward velocity between t = 2A/cand t = 3A/c before damping out. This is due to the large negative pressure reflected back from the air-backed plate. The time history of the pressure on the front and back of plate 1 (Fig. 3b) follows almost the same pattern as the water-backed case until shortly after t = 2A/c when a large negative pressure on the back of plate I occurs which leads in time the negative pressure on the front. This condition is responsible for the second forward surge of plate I in the air-backed case. The front and back pressure curves cross, as in the water-backed case, when the peak velocities occur, either positive or negative. The time history of the pressure on the front and back of plate 2 is shown in Figure 3c. The pressure on the back of plate 2 is set to 0. This condition forces the pressure difference across plate 2 to be much greater than when it was water-backed; consequently, plate 2 reaches a much greater velocity.

Figure 42 shows the plate velocities where the plate masses are 392 kg/m and plate 2 is water-backed. The velocity peaks are smaller and broader due to the larger inertia of the plates. Figure 5a shows the plate velocities where the first plate has a mass of 124 kg/m and the second plate has a mass of 392 kg/m and is air-backed. As is expected the second plate attains a velocity higher than the 392 kg/m water-backed plate but lower than the 124 kg/m air-backed plate. The first plate has a larger negative velocity peak but a smaller second positive velocity peak than in the case where the second plate was 124 kg/m and air-backed (Figure 3a). This occurs because the reflected wave from the heavier plate has a greater positive pressure peak but small negative pressure peak than the reflected wave from the lighter plate. This can be seen by comparing the pressures on the back of plate 1 shown in Figures 3b and 5b. Figure 5c shows the pressure on the front and back of plate 2.

LUMPED PARAMETER MODEL OF FLUID-PLATE SYSTEM

A second model of the double plate problem consists of 2 plates and the fluid region in between modeled as a lumped parameter system of n + l masses and n springs. A similar model has been used in calculating the responses of free-flooding areas of submarines to underwater explosion attacks. The water in the free flooded areas of the United Kingdom's A2 model submarine [12] was modeled using NASTRAN solid elements. The solid elements were specified to have a very small shear and bulk modulus matching that of water. This representation allowed for wave propagation in the interior fluid. The exterior fluid was modeled using a doubly asymptotic approximation whose early-time response corresponded to the flat-plate formulation. The present investigation was undertaken in part to check the appropriateness of modeling the interior fluid with solid elements as described above. Those examples where the plate masses are both 124 kg/m² and A/c is 2.1

ms model the water-filled bridge-fin (sail) of the A-2 model. The solutions in these cases represent the response of the two sides of the sail to the passage of the shock wave from some of the tests. In reference 12 no attempt was made to determine the optimum solid element size with respect to element ringing. The calculations presented in this report show the effect of varying the number of fluid elements on the plate responses.

The lumped parameter system shown in Figure 6 is governed by a system of n+1 differential equations, which can be written as:

$$(m_1 + w)\tilde{X}_1 = P_1 + P_2 + KX_2 - KX_1$$
 (16)

$$2w\ddot{X}_{2} = KX_{3} - 2KX_{2} + KX_{1}$$
 (17)

$$2wX_{n} = KX_{n+1} - 2KX_{n} + KX_{n-1}$$
 (18)

$$(m_2 + w)\ddot{x}_{n+1} = -Kx_{n+1} + Kx_n - P_7$$
 (19)

where
$$P_1 = P_0 e^{-st}$$

 $P_2 = P_1 - \rho c \dot{X}_1$ (20)

$$P_7 = \rho c \dot{x}_n + 1$$

 $w = \rho A/(2n)$ fluid mass

 $K = n\rho c^2/A$ spring constant

This system of n+1 differential equations may be solved by using a Runge-Kutta method.

Figure 7 shows the numerical solution for n=2, which corresponds to 2 fluid elements, or 3 coupled differential equations. The plate masses are 124 kg/m² and A/c is about 2.1 ms, the same parameters used in the continuous fluid case. The first plate accelerates then undergoes damped oscillations. The second plate slowly accelerates then decelerates. Figure 8 shows the numerical solution for n=4 which corresponds to 4 fluid elements or 5 differential equations. Plate 1 undergoes highly damped oscillations and plate 2 less damped oscillations. Oscillations occur in the velocity due to the compressibility of the fluid introduced through the spring constant K which is proportional to the bulk modulus of the fluid, pc^2 . The first plate's oscillations are more highly damped than the second plate's because the

plate 1 is loaded on one side by the pressure pulse and on the other side by the spring system, the force balance tending to reduce the amplitude of the velocity oscillations. Plate 2 is loaded only on one side by the transmitted pressure wave traveling through the spring system. Figure 9 shows the same calculation for 5 fluid elements. The oscillations are slightly higher in frequency. Figure 10, for 20 fluid elements shows still higher frequency oscillations. The time histories for the plate including the small negative velocity spike of plate I due to the reflected wave is similar to the continuous fluid case (Figure 2a). Figure 10b shows the time histories of the forces on the front and back of plate 1. The force on the front of plate 1 leads in time the force on the back, but when the reflected wave returns the force on the back leads the force on the front. The force amplitudes oscillate but cross at the times when the peak plate velocities occur. Figure 10c shows the time histories of the forces on the front and back of plate 2. The force on the front of plate 2 leads in time the force on the back, the two crossing at times corresponding to the peaks and nulls in the plate velocity oscillations. The force time histories resemble the continuous fluid pressure time histores (Figures 2b and 2c) except for the damped oscilla-It is evident that as more fluid elements are added that the lumped parameter solution approaches the continuous fluid solution. Even the crude model with 2 fluid elements yields peak plate velocities, which have similar magnitudes and times to the continuous fluid case. Figures 11a, b, and c show the calculation for 20 fluid elements, but where the second plate is air-backed. The times histories of the plate velocities and forces on the plates resemble the continuous fluid case (Figures 3a, b, and c). Finally, figures 12a, b and c which are for 20 fluid elements, plate 1 of mass 124 kq/m^2 , and plate 2 of mass 392 kq/m^2 and air-backed, look similar to the analogous continuous fluid results (Figures 5a, b, and c).

CONCLUSIONS

The cases where the plates are of equal mass and the second plate is water-backed most closely model the sail appendage on a submarine. The most important conclusion from the water-backed cases is that the second plate accelerates as if the first plate were not there or in other words the first plate is transparent to the pressure pulse. The small negative velocity spike of the first plate due to the reflected wave is the only modification to the plate motion caused by the fluid coupling.

The cases where the second plate is air-backed also show that the first plate is almost transparent to the pressure pulse. The only coupling effect different from the water-backed case is the reflected pressure wave with its negative pressure causing the first plate to experience a second forward velocity surge shortly after t=2A/c.

Changing the masses of the plates only affects the amplitude and width of the velocity and pressure peaks but introduces no major qualitative variations in the time histories.

For the range of parameters invastigated (which include an approximate correspondence with the water-filled sail of the A2 model) modeling interior fluid as an undamped system of lumped springs and masses degrades the detail of the calculated motion by introducing damped oscillations, but the general features of the response (timing and magnitude of the peak velocities, for example) are not greatly affected even for relatively crude representations of the fluid, of 2 to 5 fluid elements. As more fluid elements are used, for example, 20, the time histories of the plate velocities and forces approach the continuous fluid time histories.

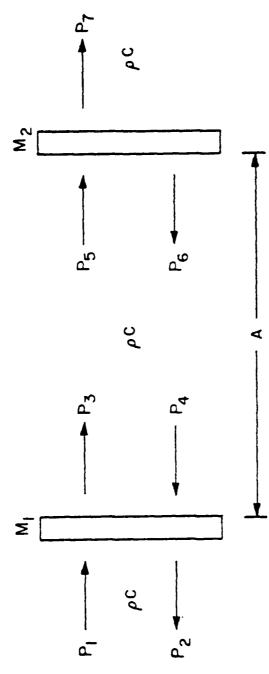


Fig. 1 — Model of a plane wave pressure pulse (shockwave) normally incident on a pair of flat plates in water. The incident shockwave (P_1) enters from left and generates reflected and transmitted waves labeled as P_2 through P_7 .

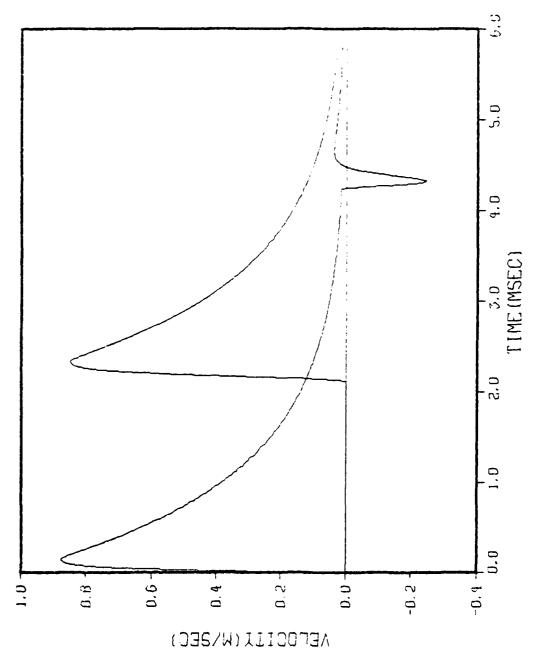
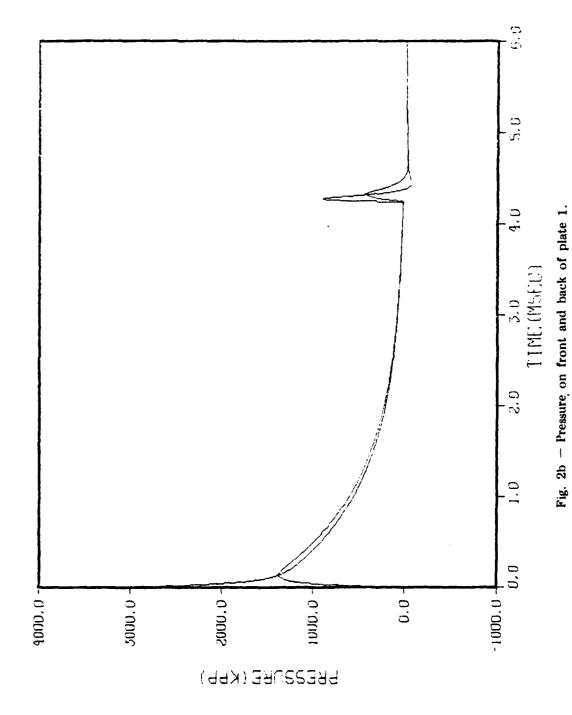
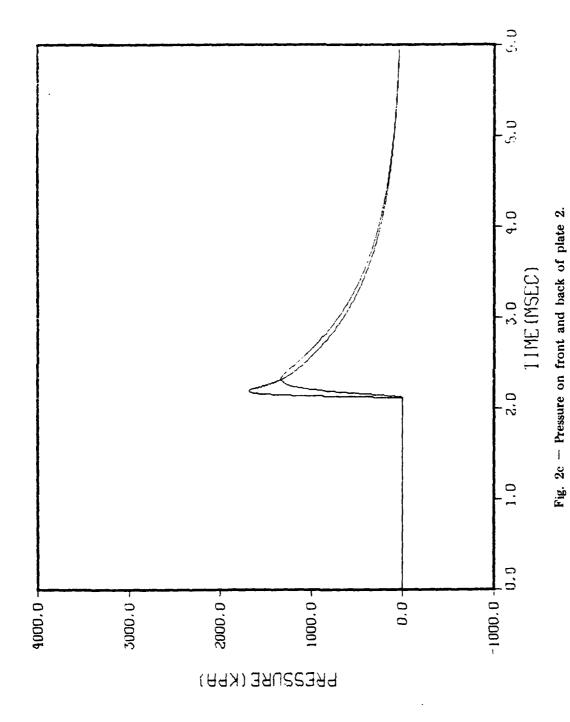


Fig. 2a — Velocity of plates immersed in water (second plate is waterbacked). Plates are of mass 124 kg/m² and separated by an A/C of 2.1 milliseconds. The shockwave has a peak particle velocity of 1m/sec and decays in amplitude by 1/e in 1 millisecond.





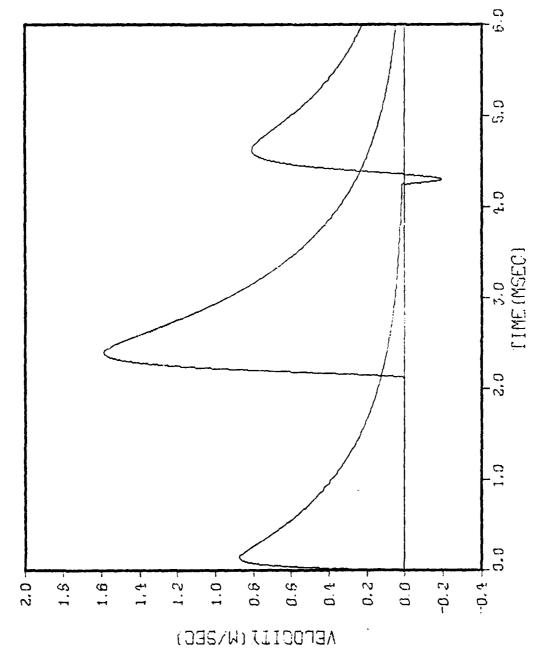
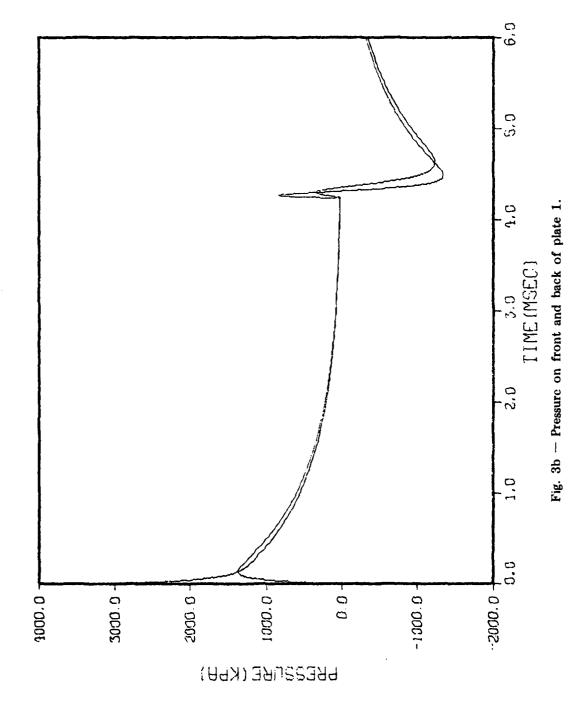
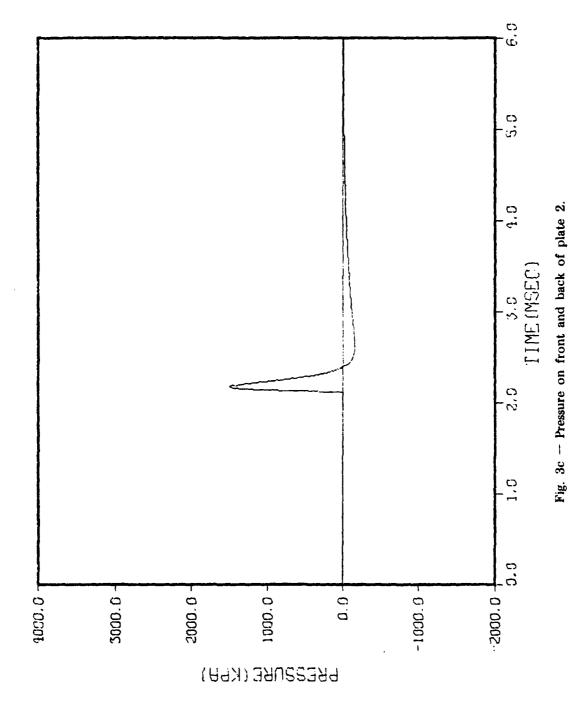


Fig. 3a - Velocities of plates when second plate is air-backed. The plate masses are 124 kg/m².





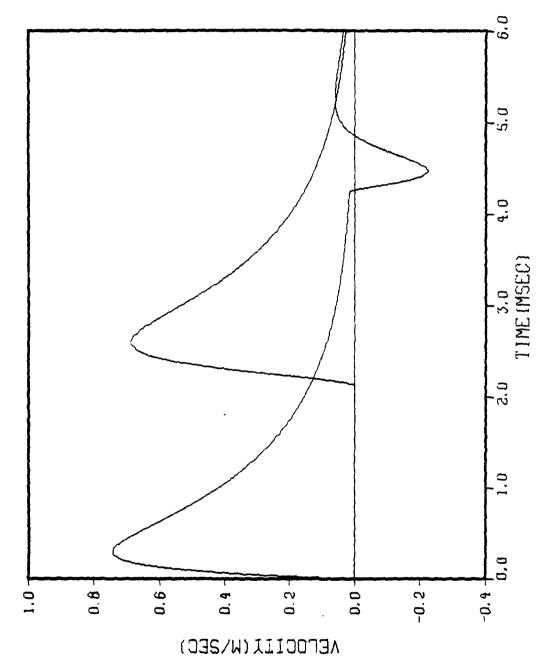


Fig. 4 \sim Velocities of the plates when the plate masses are 392 kg/m² and the second plate is water backed.

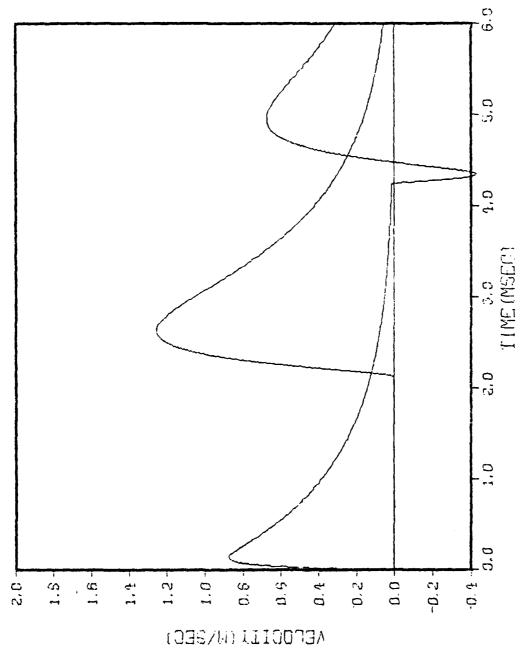
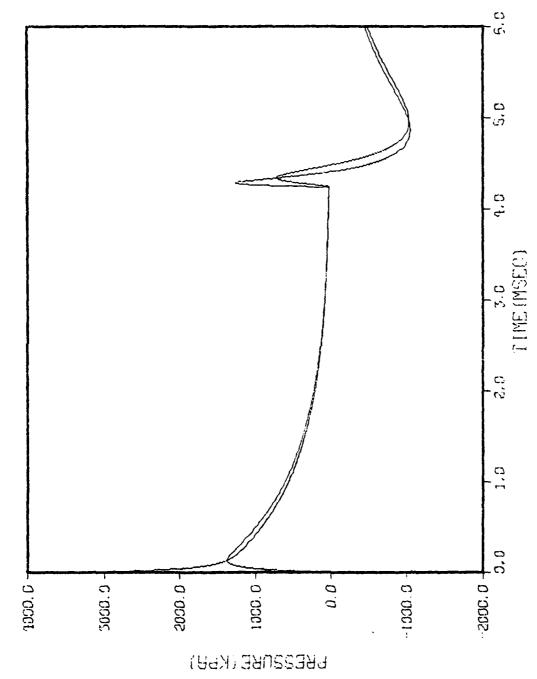
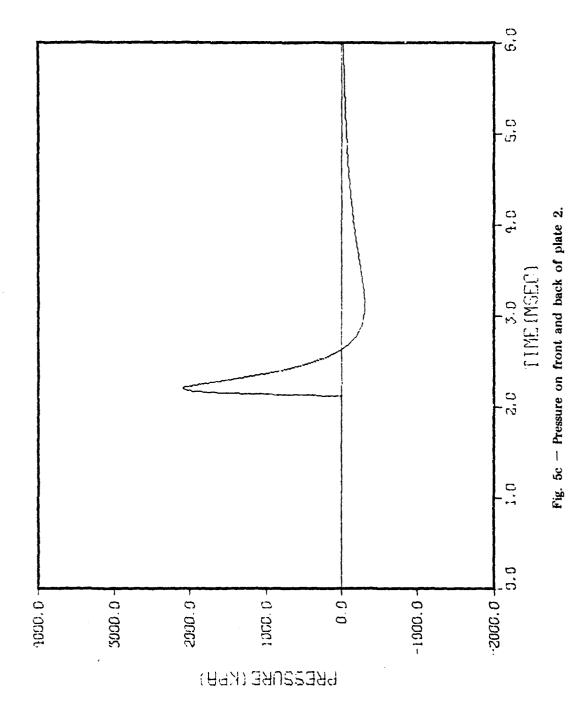


Fig. 5a — Velocities of plate when plate 1 has mass $124~{\rm kg/m^2}$ and plate 2 has mass $392~{\rm kg/m^2}$ and is air-backed.





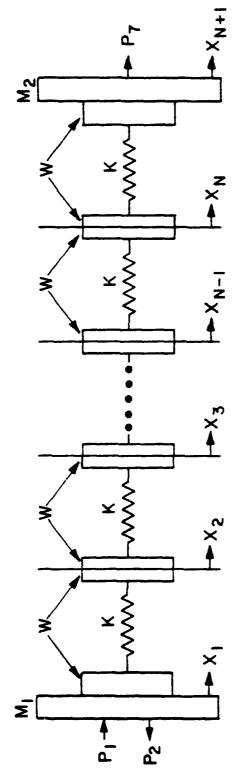


Fig. 6 — Model of a plane-wave pressure pulse normally incident on a pair of flat plates with the fluid in between modeled by a set of lumped masses and springs. The incident shockwave (P_1) enters from the left, gives rise to a reflected wave (P_2) , a wave traveling through the mass-spring system, and a transmitted wave P_7 leaving the back of plate 2.

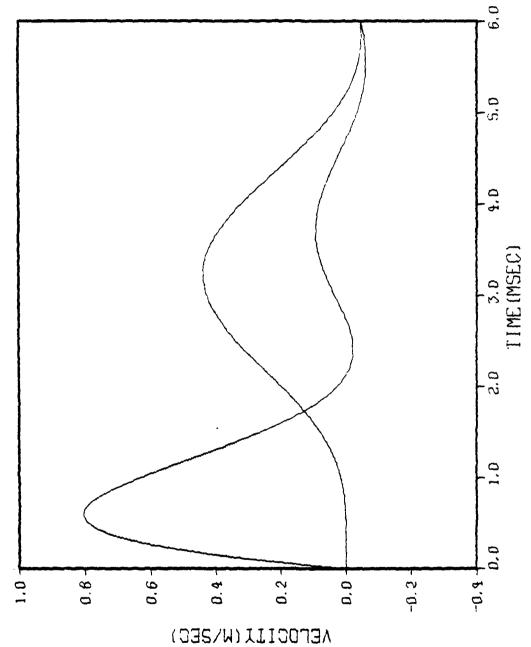


Fig. 7 — Plate velocities when the interior fluid is modeled with 2 fluid elements. The plate masses are both $124~{\rm kg/m^2}$ and the second plate is water-backed.

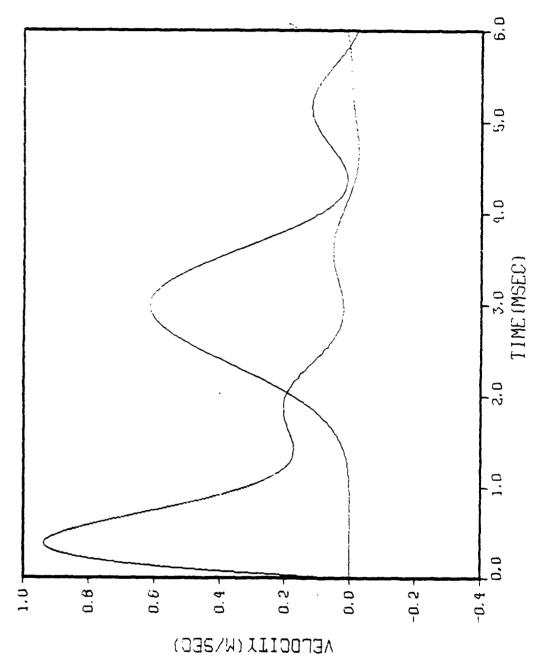


Fig. 8 — Plate velocities when the interior fluid is modeled with 4 fluid elements. The plate masses are both $124~{\rm kg/m^2}$ and the second plate is water-backed.

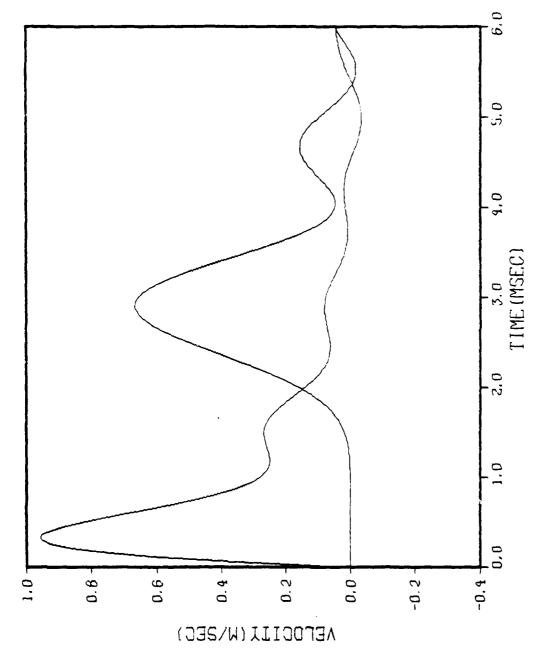


Fig. 9 — Plate velocities when the interior fluid is modeled with 5 fluid elements. The plate masses are $124~{\rm kg/m}^2$ and the second plate is waterbacked.

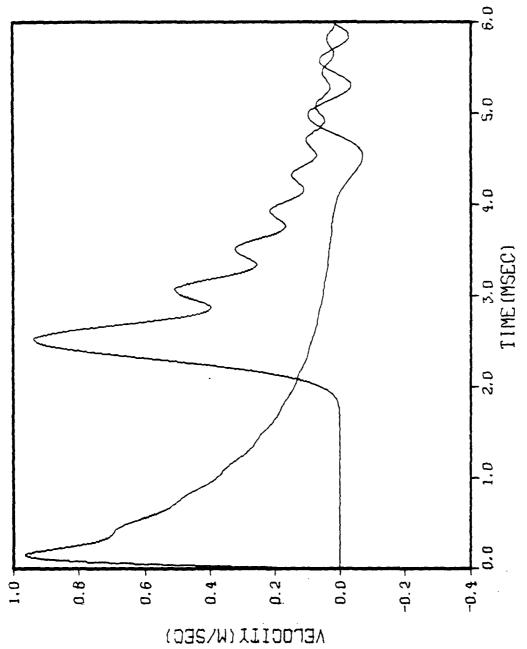
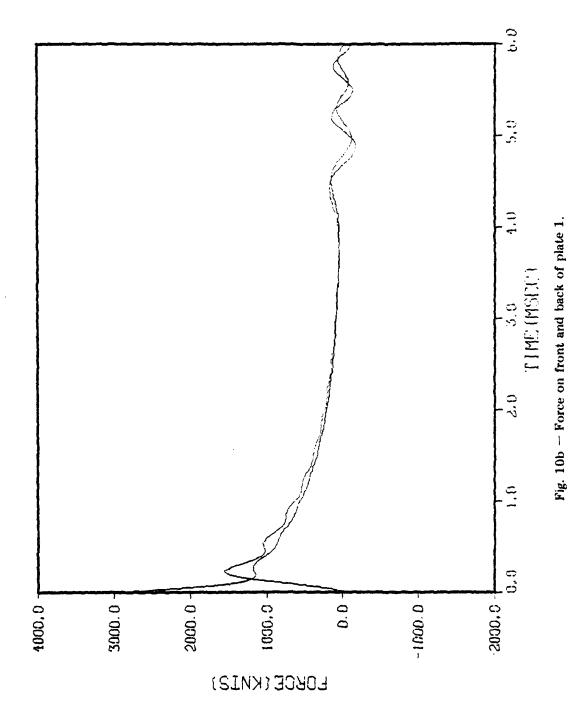
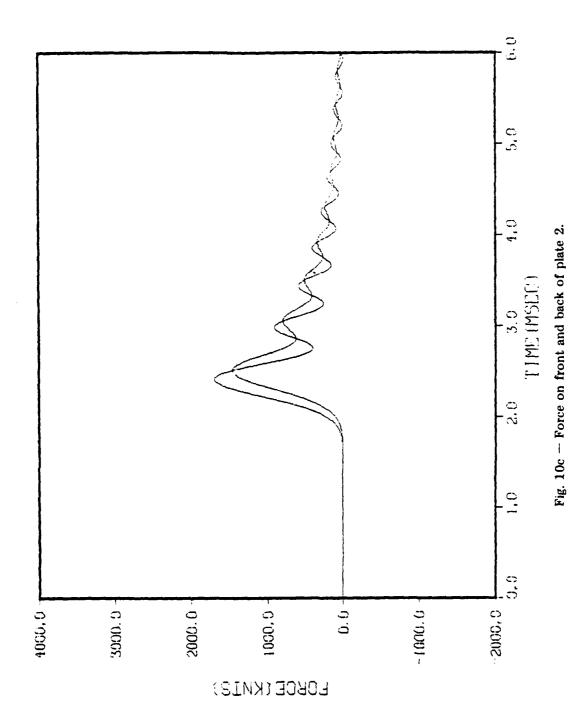


Fig. 10a — Plate velocities when the interior fluid is modeled by 20 fluid elements. The plate masses are 124 kg/m 2 and the second plate is waterbacked.





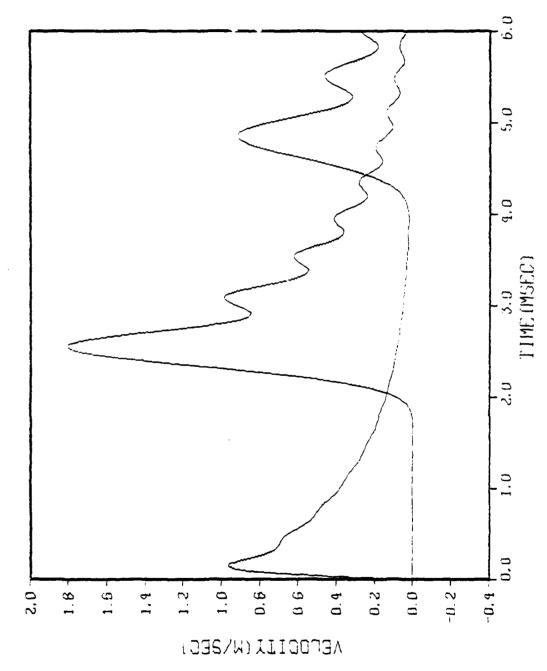


Fig. 11a – Plate velocities when the interior fluid is modeled by 20 fluid elements. Plate 1 has mass 124 kg/m², plate 2 has mass 124 kg/m², and is air-backed.

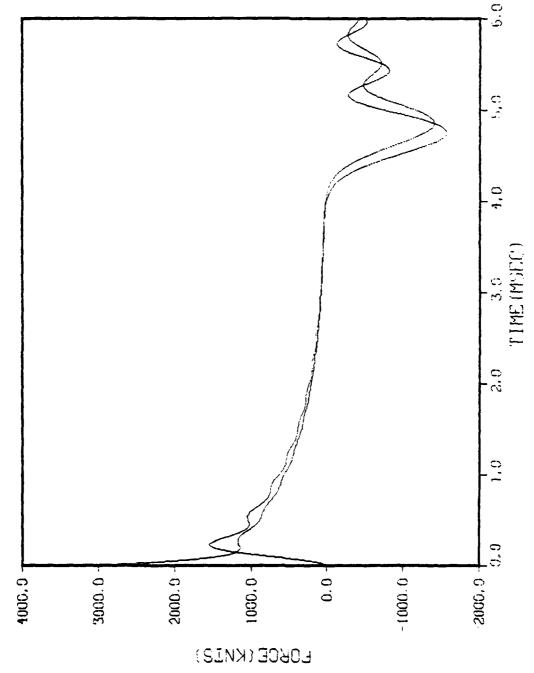
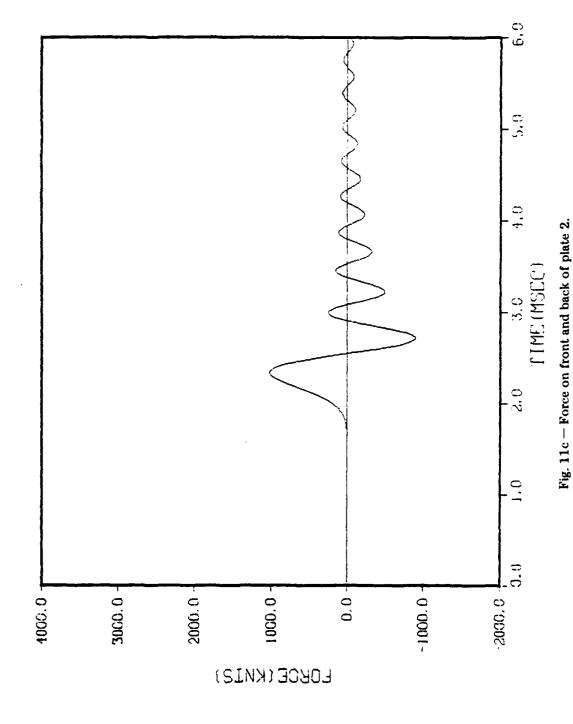


Fig. 11b - Force on front and back of plate 1.



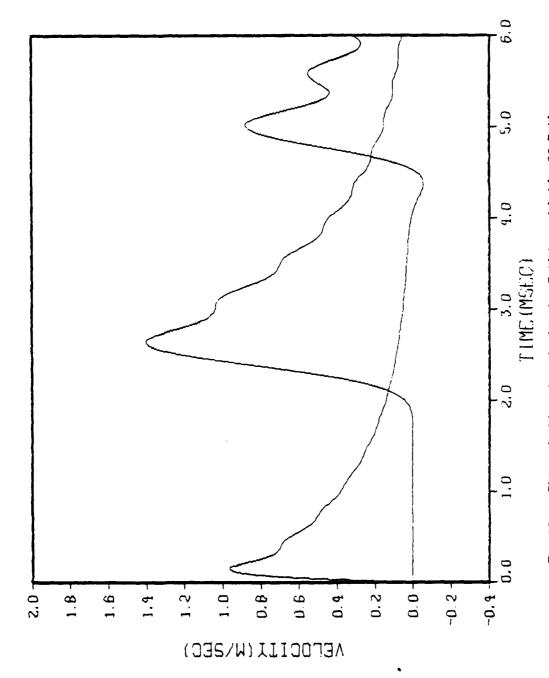
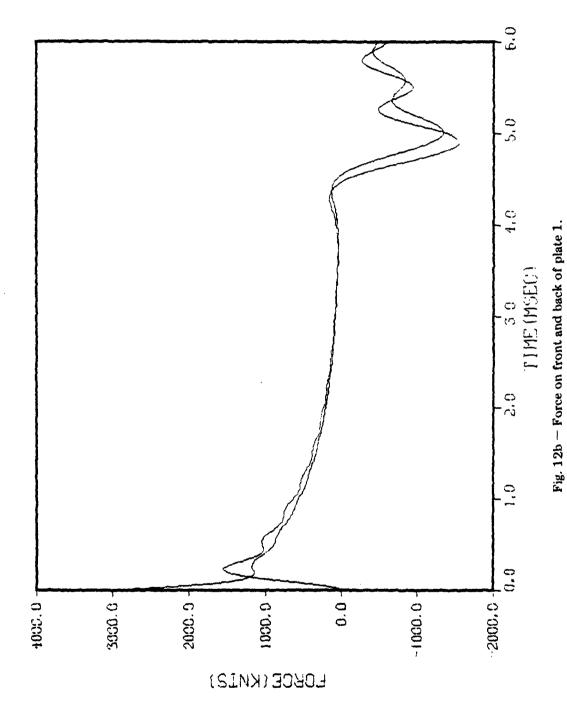
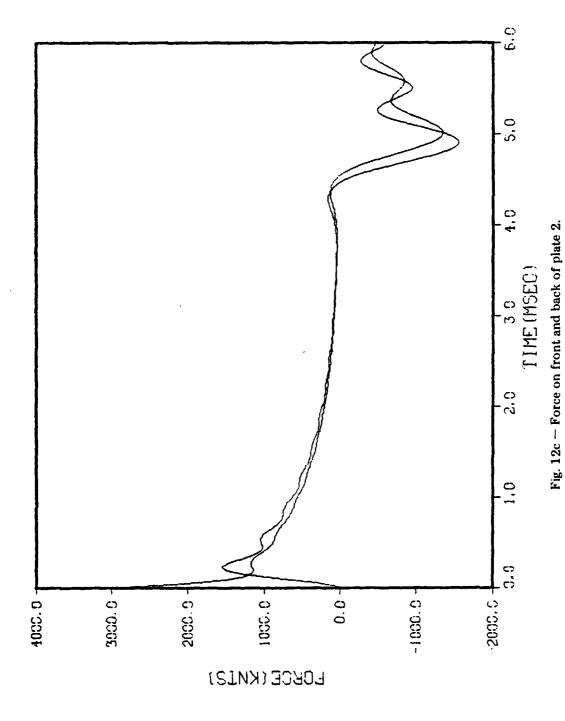


Fig. 12a — Plate velocities when the interior fluid is modeled by 20 fluid elements. Plate 1 has mass 124 kg/m², plate 2 has mass 392 kg/m² and ι . airbacked.





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